A Methodological Note on the Estimation of Programming Models

by

Thomas Heckelei and Hendrik Wolff[1]

TWP-2001-104

Corresponding Author: Thomas Heckelei

Institute for Agricultural Policy, Market Research and Economic Sociology, University of Bonn

Nussallee 21, 53115 Bonn

Tel.: +49-228-732324

E-mail: heckelei@agp.uni-bonn.de

[1] Thomas Heckelei, Hendrik Wolff, Institute for Agricultural Policy, Market Research and Economic Sociology of Bonn University, Nussallee 21, 53332 Bonn, Germany; email: heckelei@agp.uni-bonn.de; wolff@agp.uni-bonn.de.

A Methodological Note on the Estimation of Programming Models

Abstract

The paper introduces a general methodological approach for the estimation of constrained optimisation models in agricultural supply analysis. It is based on optimality conditions of the desired programming model and shows a conceptual advantage compared to Positive Mathematical Programming in the context of well posed estimation problems. Moreover, it closes the empirical and methodological gap between programming models and duality based functional models with explicit allocation of fixed factors. Monte Carlo simulations are performed with a maximum entropy estimator to evaluate the functionality of the approach as well as the impact of empirically relevant prior information in small sample situations.

Keywords

Agricultural Supply Analysis, Programming Models, Maximum Entropy Estimation, Prior Information

1 Introduction

Quantitative models of multi-output multi-input crop supply behaviour in agriculture typically belong to one of two main methodological types: either programming models or dual systems of supply and input demand equations. The former determine input allocation to various production activities using an explicit optimisation, the latter constitute analytical solutions to economic optimisation models. Maintained economic hypotheses and objectives do not necessarily have to differ between these types.[2] However, in empirical reality the structure and specification procedures are clearly distinguished: A programming model is chosen when the analyst sees the need to explicitly model complex technological or political constraints under which behavioural functions cannot be derived easily or at all. This generally comes at the cost of lacking statistical estimation and validation for the whole model. Dual equation systems, on the other hand, allow to apply well established econometric techniques to base the parametric specification on observed supply and input demand decisions of agricultural producers. This choice limits the model's complexity and potentially oversimplifies for the purpose of a differentiated analysis.

During the last decade both type of methodological approaches seemingly moved a little closer to each other. Just and Chambers (1989) developed a dual supply model specification with explicit allocation of fixed factors. This allowed to overcome a previous deficiency for modelling agricultural crop supply by incorporating land constraints and the observable decision variable 'land allocated to production activities'. It also provided a useful framework to model the European policy instrument 'hectare premium' distinct from product price effects (Guyomard et al., 1996; Moro and Schokai, 1999). Nevertheless, additional constraints cannot easily be incorporated and the choice of functional form is restricted due to analytical limitations. From the programming side, Howitt (1995a) presented 'Positive Mathematical Programming' (PMP) which allows to calibrate models to observed behaviour of a base year. PMP established itself as the dominant approach for the specification of programming models designed for policy analysis (for example: Howitt and Gardner 1986; House 1987; Kasnakoglu and Bauer 1988; Horner et al. 1992; Schmitz 1994; Arfini and Paris 1995; Barkaoui and Butault

---

[2] Often, programming models are characterised as 'normative' in general, because they use an explicit optimisation. This neither reflects the original meaning nor is it a very useful distinction. The objective of normative analysis is to say 'what should be' and in this respect farm- or regional *planning* models qualify for this category. Programming models designed to explain or project behaviour do not. An integratable 'positive' dual supply system could just as well be used as an explicit optimisation model for simulation and yield the exact same results.

1999; Cypris 2000; Graindorge et al. 2001; Helming et al. 2001). The incorporation of several observations employing an econometric criterion was generally made possible by Paris and Howitt (1998) and put to work for a cross sectional data set by Heckelei and Britz (2000). However, the theoretical base of this approach is weak or at least veiled.

This paper aims at further moving the two methodological approaches closer together. We present a general approach to estimate parameters of programming models for agricultural supply analysis based on optimality conditions of the desired model. The method provides a consistent alternative to PMP and allows to estimate models with multiple constraints which do not allow to analytically solve for behavioural functions. The paper is organised as follows: the next section explains why PMP is not well suited for the estimation of programming models based on multiple observations. Section three describes a general alternative. The main section four illustrates the approach for three different optimisation models which stem from the programming and econometric literature. It provides Monte Carlo simulation results to demonstrate functionality with pure data based estimates. In addition, approaches using prior information exploit the potential of maximum entropy techniques in this context and address the problem of limited sample sizes often confronted by differentiated modelling exercises. Section five concludes and points at promising directions of further research.

2 Positive Mathematical Programming: Short Review and Critique

The general idea of PMP is to employ dual values of calibration constraints which force the optimisation model to observed outcomes of endogenous variables (step 1). These dual values are used to specify additional non-linear terms in the objective function which allow to reproduce the observed outcomes exactly without calibration constraints (step 2). Starting from a typical linear program (LP) in agricultural supply analysis step 1 can be illustrated as

(1)
$$\begin{aligned} &\max_{\mathbf{l}} Z = \mathbf{p}'\mathbf{l} - \mathbf{c}'\mathbf{l} \\ &\text{subject to} \\ &\mathbf{Al} \le \mathbf{b}\ \ [\boldsymbol{\lambda}],\ \ \mathbf{l} \le \left(\mathbf{l}^{o} + \boldsymbol{\varepsilon}\right)\ [\boldsymbol{\rho}],\ \ \mathbf{l} \ge \mathbf{0} \end{aligned}$$

where Z is the objective function value, **p**, **l**, and **c** are (N×1) vectors of product prices, non-negative activity levels, and variable costs per activity unit, respectively. **A** represents a (M×N) matrix of coefficients, **b** and $\boldsymbol{\lambda}$ are (M×1) vectors of resource availability and their corresponding shadow prices. The (N×1) vector $\mathbf{l}^{o}$ are observed activity levels in a base period, $\boldsymbol{\varepsilon}$ is a (N×1) vector of small numbers and $\boldsymbol{\rho}$ (N×1) contains the dual variables of the calibration constraints. In the second step of PMP the dual values $\boldsymbol{\rho}$ are used to specify a non-linear variable cost function $C^{V}(\mathbf{l}^{o})$, such that the 'variable' marginal cost $\mathbf{MC}^{V}(\mathbf{l}^{o})$ of the activities are equal to the sum of the known cost **c** and the 'non-specified marginal cost' $\boldsymbol{\rho}$. In case of the frequently used quadratic functional form the following condition for calibration is implied:

$$(2) \qquad \mathbf{MC}^{V} = \frac{\partial C^{V}(\mathbf{l}^{o})}{\partial \mathbf{l}} = \mathbf{d} + \mathbf{Q}\mathbf{l}^{o} = \mathbf{c} + \boldsymbol{\rho},$$

where the (N×1) vector **d** and the (N×N) symmetric positive definite matrix **Q** correspond to the linear and quadratic terms of $C^{V}(\mathbf{l}^{o})$, respectively. This condition does not include the opportunity cost of using fixed resources, because those are still accounted for by the resource constraints in the resulting model

$$(3) \qquad \begin{aligned} &\max_{\mathbf{l}} Z = \mathbf{p}'\mathbf{l} - \mathbf{d}'\mathbf{l} - 0.5\mathbf{l}'\mathbf{Q}\mathbf{l} \\ &\text{subject to} \\ &\mathbf{A}\mathbf{l} \leq \mathbf{b} \ \left[\boldsymbol{\lambda}\right], \ \ \mathbf{l} \geq \mathbf{0} \end{aligned} .$$

In order to solve the underdetermined system (2) with N+N(N+1)/2 parameters and N equations, the literature suggests many approaches which include simple ad-hoc procedures with some parameters set a-priori (for example Howitt, 1995a), the use of supply elasticities (Helming et al. 2001), and the employment of a maximum entropy criterion (Paris and Howitt 1998). As long as conditions (2) are satisfied, the calibration of the resulting optimisation model is guaranteed, but the different specification of **d** and **Q** imply significant differences with respect to the simulation behaviour (see Cypris 2000 or Heckelei and Britz 2000).

However, in this paper not calibration but *estimation* of programming models is of major concern. Paris and Howitt (1998 and 2001) already point at the possibility that more than one observation on production programs could be incorporated, providing a set of N marginal cost conditions (2) for each observation. Heckelei and Britz use this idea for the estimation of regional cost functions based on a cross sectional sample.

Here we want to argue that the PMP-procedure is not well suited to exploit the additional data information, because the derived marginal cost conditions do not allow to consistently estimate the parameters. For this purpose it is useful to look at PMP from the perspective of an econometrician: This implies to have some idea on a 'true' model, or at least the assumption, that a specific model is capable to sufficiently represent the true data generating process. A multitude of PMP-modellers apparently believed this with regard to the resulting non-linear model ultimately used to perform economic analysis. If we take, for example, the quadratic model (3) and assume exclusively positive activity levels and binding resource constraints at the optimal solution, the Kuhn Tucker conditions imply the shadow price values $\boldsymbol{\lambda} = \left(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}'\right)^{-1}\left(\mathbf{A}\mathbf{Q}^{-1}\left(\mathbf{p}-\mathbf{d}\right)-\mathbf{b}\right)$.

In the first step of PMP, however, a different result is obtained: we partition **l** into two subvectors, a ((N-M)×1) vector of 'preferable' activities, $\mathbf{l}^p$, bounded by the calibration constraints and a (M×1) vector of marginal activities, $\mathbf{l}^m$, bounded by the resource limits. Then the dual values

$$\text{(4)} \qquad \text{(a) } \boldsymbol{\lambda} = \left(\mathbf{A}^{m}{}'\right)^{-1}\left(\mathbf{p}^{m} - \mathbf{c}^{m}\right), \qquad \text{(b) } \boldsymbol{\rho}^{p} = \mathbf{p}^{p} - \mathbf{c}^{p} - \mathbf{A}^{p}{}'\boldsymbol{\lambda}, \qquad \text{(c) } \boldsymbol{\rho}^{m} = \mathbf{0},$$

can be derived. We can see that **λ** is exclusively determined by objective function entries and technological coefficients of the marginal activities and are therefore generally different from the values of the quadratic model. Since step 1 of PMP sets **ρ** simultaneously with **λ** (4b) and step 2 uses **ρ** to specify $\mathbf{MC}^V$, the latter are generally biased. Consequently, the set of equations (2) cannot be seen as unbiased estimating equations and will generally yield inconsistent parameter estimates if the true data generating process is correctly described by the quadratic model.

3 A General Alternative

The suggested 'general alternative' to PMP relies on a simple principle. It directly employs the optimality conditions of the desired programming model. No 'step 1' for the determination of dual values of calibration constraints is necessary. Instead, the simultaneous estimation of shadow prices and parameters avoids methodological inconsistencies.

The basic principle can be illustrated by writing the programming model as a general Lagrangian form with an objective function $h(\mathbf{y}|\boldsymbol{\alpha})$ to be optimised subject to a constraint vector $\mathbf{g}(\mathbf{y}|\boldsymbol{\beta}) = 0$:

$$L(\mathbf{y},\boldsymbol{\lambda}|\boldsymbol{\alpha},\boldsymbol{\beta}) = h(\mathbf{y}|\boldsymbol{\alpha}) + \boldsymbol{\lambda}'[\mathbf{g}(\mathbf{y}|\boldsymbol{\beta})],$$

where **y**, **λ**, **α**, and **β** represent column vectors of endogenous variables, unknown dual values, parameters of the objective function, and parameters of the constraints, respectively. The appropriate first order optimality conditions are the gradients with respect to **y** and **λ** set to zero:

$$\frac{\partial L}{\partial \mathbf{y}} = \frac{\partial h(\mathbf{y}\,|\,a)}{\partial \mathbf{y}} + 1'\frac{\partial \mathbf{g}(\mathbf{y}\,|\,b)}{\partial \mathbf{y}} = \mathbf{0}$$

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{g}(\mathbf{y}\,|\,\boldsymbol{\beta}) = \mathbf{0}.$$

For the case of inequality constraints $\mathbf{g}(\mathbf{y}|\boldsymbol{\beta}) \leq 0$ we need to substitute the gradient with respect to **λ** by the complementary slackness representation[3]

[3] The symbol ' ' represents the Hadamard or element-wise product of two matrices. If a_{ij} and b_{ij} are the elements of two matrices with equal dimension, **A** and **B**, then **A** **B** = **C**, where **C** is of the same dimension as **A**, **B** and each element of **C** is defined as $c_{ij} = a_{ij} \cdot b_{ij}$.

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{g}(\mathbf{y} \mid \boldsymbol{\beta}) \leq \mathbf{0}; \quad \boldsymbol{\lambda} \quad \mathbf{g}(\mathbf{y} \mid \boldsymbol{\beta}) = \mathbf{0}$$

The unknowns **λ**, **α**, and **β** of these Kuhn Tucker conditions can be estimated with some econometric criterion directly applied to these equations. Depending on the parametric specification appropriate curvature restrictions (second order conditions) might have to be enforced as well.

The direct use of optimality conditions for estimation is certainly not new by itself. In the context of investment models, for example, the dynamic equivalents of Kuhn Tucker conditions, the Euler equations, have been frequently used as estimating equations to overcome analytical and empirical problems for more complex models (Chirinko 1993:1893f.). However, their employment as an alternative to PMP or to the estimation of behavioural functions in the context of multi-output agricultural supply models has not been considered. One of the examples in the subsequent section will show that this approach is not only useful for the estimation of typical agricultural programming models but also provides a flexible alternative for estimating parameters of duality based behavioural functions with explicit allocation of fixed factors. In this context, the only difference left between programming and econometric models is the model form used for simulation purposes.

It is perhaps not surprising that the most innovative PMP proponents Paris and Howitt (2001) already used this principle for the calibration of an agricultural supply model. Their ‘Symmetric Positive Equilibrium Problem’ calibrates a multi-input multi-output profit maximisation model on the basis of the corresponding marginal cost conditions. However, the authors did not realise that their 'first phase' employed to determine dual values of calibration constraints is irrelevant for the approach. In addition, our examples in the next section differ from Paris and Howitt such that the presented models all imply the existence of at least one fixed factor. They are supposed to illustrate the suggested principle for the estimation of programming models. Monte Carlo simulations with (Generalised) Maximum Entropy estimates shall indicate the consistency of the approach and allow to assess the influence of prior information on estimation results in situations with limited data information. The featured programming models are not necessarily the most useful models for agricultural supply analysis, but are rather chosen to span the literature on programming models and econometric models with explicit allocation of fixed factors.

4 Examples and Monte Carlo Evidence

4.1 Land Allocation with Quadratic Cost Function

This subsection deals with estimating the parameters of the optimisation model employing a quadratic cost function often used in the PMP context and already described above. For the sake of simplicity we consider just the resource land as fixed rendering a quadratic programming model (QP-model) with a scalar shadow price. In addition, we replace the vector of prices **p** by a vector of gross margins **gm**[4] to obtain

$$(5)\qquad \begin{array}{l} \max_{\mathbf{l}} Z = \mathbf{gm}'\mathbf{l} - \mathbf{d}'\mathbf{l} - 0.5\mathbf{l}'\mathbf{Q}\mathbf{l} \\ \text{subject to} \\ \mathbf{u}'\mathbf{l} \le b \;\; [\lambda], \;\; \mathbf{l} \ge \mathbf{0} \end{array},$$

with the (N×1) summation vector **u**, i.e. a vector of ones.

If we assume that the optimal land allocations satisfy the land constraint in equality form for every observation t = 1,...,T, and that observed land allocations, $\mathbf{l}_t^o$, are obtained from optimal values by adding an (N×1) vector of stochastic errors $\mathbf{e}_t$ with mean zero and standard deviation σ_i, we can write the first order conditions as[5]

$$(6)\qquad \begin{array}{l} \mathbf{gm}_t^o - \lambda_t \mathbf{u} - \mathbf{d} - \mathbf{Q}\left(\mathbf{l}_t^o - \mathbf{e}_t\right) = 0 \\ \mathbf{u}'\left(\mathbf{l}_t^o - \mathbf{e}_t\right) = b_t^o \end{array}.$$

As an estimation technique, we employ the Generalised Maximum Entropy GME approach based on Golan et al. (1996), which allows a flexible incorporation of out of sample information.[6] We reparameterise the error vectors as expected values of a discrete probability distribution. The (N×(N·2)) Matrix **V** with S=2 support points for each error term bounds the support to +/-5 standard deviations.[7] For the simulation experiments below we have N=3 crops so that the error terms can be represented as the multiplication of **V** with a ((N·S)×1)-vector of probabilities $\mathbf{w}_t$ to obtain

---

[4] The quadratic cost function represents ‘some’ unknown non-linear cost, which are independent of the variable inputs per activity unit. This lack of rationalisation in the model is analogous to many PMP applications. The illustration based on the model shall by no means indicate the preferability of this model.

[5] Here and subsequently do we employ the notational convention that equations are valid for all elements in the respective indices, i.e. in this specific case for all t = 1,...,T.

[6] It shall be mentioned, however, that in this context of ‘well-posed’ estimation problems with more observations than parameters to be estimated, classical techniques such as least squares could have been applied as well.

[7] The ‘right’ number of support points as well as the range of the support is an often discussed but not ultimately solved question. We chose 2 two support points here mainly to restrict the computational demands in the already complex Monte Carlo simulation exercises, despite the fact that 3 or 4 support points promise a limited reduction of the mean estimation error (Golan et al. 1996:139-40). With respect to the support range Golan et al. (1996) suggest the ‘3-Sigma’ rule. Preckel (2001) advocates a rather large range to approximate the behaviour of the least squares estimator.

$$(7)\qquad \mathbf{e}_t = \mathbf{V}\mathbf{w}_t = \begin{bmatrix} -5\sigma_1 & 5\sigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5\sigma_2 & 5\sigma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5\sigma_3 & 5\sigma_3 \end{bmatrix} \begin{bmatrix} w_{11t} \\ w_{12t} \\ w_{21t} \\ w_{22t} \\ w_{31t} \\ w_{32t} \end{bmatrix}.$$

The complete GME formulation is then[8]

$$(8)\qquad \max_{\mathbf{w}_t,\mathbf{Q},\mathbf{L},\lambda_t} H(\mathbf{w}_t) = -\sum_{t=1}^{T} \mathbf{w}_t{}'\ln \mathbf{w}_t$$

subject to

$$(9)\qquad \mathbf{gm}_t^o - \lambda_t \mathbf{u} - \mathbf{Q}\left(\mathbf{l}_t^o - \mathbf{V}\mathbf{w}_t\right) = 0$$

$$(10)\qquad \mathbf{u}'\left(\mathbf{l}_t^o - \mathbf{V}\mathbf{w}_t\right) = b_t^o$$

$$(11)\qquad \mathbf{Q} = \mathbf{L}\mathbf{L}' \text{ mit } L_{ij} = 0 \;\forall\; j > i$$

$$(12)\qquad \sum_{s=1}^{S} w_{its} = 1$$

where $H(\mathbf{w_t})$ denotes Entropy, equation (11) guarantees the positive definiteness of **Q** based on a Cholesky factorisation, and (12) ensures that the probabilities add up to one. Note, that we do not need any reparameterisation of model parameters, because we only consider 'well-posed' problems with positive degrees of freedom in our simulations.

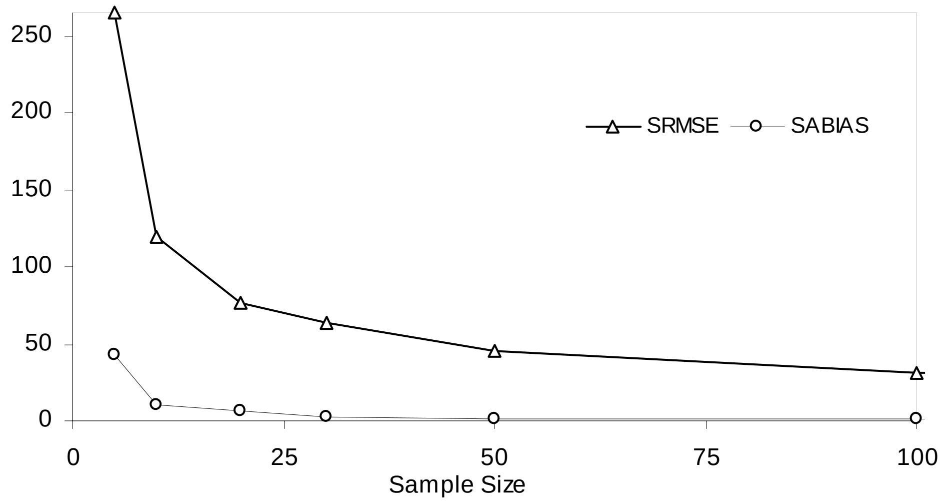
The following Monte Carlo simulation experiment is used to test the estimators precision: Based on the output and input differentiation in Howitt (1995b)[9] a data set with T observations is generated for T different random vectors $\mathbf{gm}_t$ and $\mathbf{b}_t$ for given parameters **Q**.[10] Normally distributed errors are added to the optimal land allocations l_{1t}^* and l_{2t}^* of the first two crops with a standard deviation of 2% of the average land allocation, so that the 'observed' allocations are calculated as $l_{1t}^o = l_{1t}^* + e_{1t}$ und $l_{2t}^o = l_{2t}^* + e_{2t}$. To ensure that the land restriction is binding at the observed production activity levels we let $l_{3t}^o = b_t - l_{1t}^* - l_{2t}^*$.

[8] For the current case of just one resource constraint the vector **d** is not identified. Therefore its elements are set to zero.

[9] See the appendix for the basic data set from Howitt which reflects the values of non-random model variables and the means of random variables in the simulation exercises.

[10] For the parameter values of the true QP-model model as well as the models in the following subsections see the appendix. Additional parametric restrictions included during estimation in order to obtain a well defined production function are reported in the appendix as well.

Figure 1 QP-model – SRMSE and SABIAS without prior information



Source: Own calculations.

For every generated data set, the model parameters are estimated with the GME approach and the whole procedure is repeated 1000 times for each sample size. The quality of the estimation is evaluated using the measures absolute bias ('ABIAS' = absolute value of the difference between average estimate and true value of the parameter) and root mean square error ('RMSE'). For a representative look at the results the measures are summed over all estimated parameters (here all elements of **Q**).

Figure 1 presents the results for different sample sizes. The sum of all RMSE decreases with increasing sample size indicating consistency of the estimator.[11] The bias quickly decreases to irrelevant values already at a sample size of 20 (SABIAS). Recalling that the MSE is the sum of the squared bias and the empirical variance, figure 1 shows that the bias reflects a small fraction of the RMSE only and the much more important part of the MSE is given by the standard errors of the estimates. For small sample sizes - which are often encountered in empirical work for differentiated analyses - this could obviously result in very poor estimates. In this case the use of out of sample information is a potential remedy. Ideally, the employment of prior information would reduce the estimators variance at small sample sizes without introducing a strong additional bias. To get a better feel for the required precision of the prior information and the general interplay between prior and data in our modelling context we further extended the simulations:

An empirically relevant possibility for incorporating out of sample information is the use of priors on elasticities. Other studies with comparable objectives frequently provide at least a general idea on their range. A reparameterisation of these elasticites analogous to the one for the error terms allows the technical representation. For the current model, we can employ the following analytical expression for the (N×1) vector of land allocation elasticities with respect to own gross margins $\boldsymbol{\varepsilon}$:

(13)
$$\begin{aligned}\boldsymbol{\varepsilon} &= \operatorname{diag}\left(\frac{\partial \mathbf{l}}{\partial \mathbf{gm}} \quad \left[\frac{\overline{\mathbf{gm}}}{\overline{\mathbf{l}}^{o}}\right]'\right)\\ &= \operatorname{diag}\left(\left(\mathbf{Q}^{-1}-\mathbf{Q}^{-1}\mathbf{u}\left(\mathbf{u}'\mathbf{Q}^{-1}\mathbf{u}\right)^{-1}\mathbf{u}'\mathbf{Q}^{-1}\right) \quad \left[\frac{\overline{\mathbf{gm}}}{\overline{\mathbf{l}}^{o}}\right]'\right)\end{aligned}$$

where $\left[\partial\mathbf{l}/\partial\mathbf{gm}\right]$ represents the (N×N) Jacobian matrix of the land demand functions and the i,j-th element of the (N×N) matrix $\left[\overline{\mathbf{gm}}/\overline{\mathbf{l}}^{o}\right]$ is defined as the sample mean of gross margin i, $\overline{gm_i}$, divided by the sample mean of observed land allocation to crop j, $\overline{l}_j^{o}$. Combined with the elasticity reparameterisation we have to add the constraint

(14)
$$\mathbf{V}^{\varepsilon}\mathbf{w}^{\varepsilon} = \operatorname{diag}\left(\left(\mathbf{Q}^{-1}-\mathbf{Q}^{-1}\mathbf{u}\left(\mathbf{u}'\mathbf{Q}^{-1}\mathbf{u}\right)^{-1}\mathbf{u}'\mathbf{Q}^{-1}\right) \quad \left[\frac{\overline{\mathbf{gm}}}{\overline{\mathbf{l}}^{o}}\right]'\right)$$

with

$$\mathbf{V}^{\varepsilon} = \begin{bmatrix} v_{11}^{\varepsilon} & v_{12}^{\varepsilon} & 0 & 0 & 0 & 0 \\ 0 & 0 & v_{21}^{\varepsilon} & v_{22}^{\varepsilon} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{31}^{\varepsilon} & v_{32}^{\varepsilon} \end{bmatrix} \quad \text{and} \quad \mathbf{w}^{\varepsilon} = \begin{bmatrix} w_{11}^{\varepsilon} \\ w_{12}^{\varepsilon} \\ w_{21}^{\varepsilon} \\ w_{22}^{\varepsilon} \\ w_{31}^{\varepsilon} \\ w_{32}^{\varepsilon} \end{bmatrix}$$

to the previous (9)-(12), where v_{i1}^{ε} and v_{i2}^{ε} are the lower and upper support points of the i-th elasticity, respectively, and w_{i1}^{ε} and w_{i2}^{ε} the corresponding probabilities. The objective function has to be modified to

(15)
$$\max_{\mathbf{w}_t,\mathbf{w}^{\varepsilon},\mathbf{Q},\mathbf{L},\boldsymbol{\lambda}_t} \mathrm{H}\left(\mathbf{w}_t\right) = -\sum_{t=1}^{T}\mathbf{w}_t\,'\ln\mathbf{w}_t - \mathbf{w}^{\varepsilon}\,'\ln\mathbf{w}^{\varepsilon}.$$

The intuition behind the objective function is as follows: the entropy criterion generally pulls towards the centre of the elasticity supports in trade-off with the error terms of the data constraints. The smaller the range of the elasticity support the higher is the penalty for deviating

---

[11] Mittelhammer and Cardell (2000) prove consistency of the GME approach for the general linear model under mild regularity conditions. No such general theoretical result is known to us for non-linear models except for one

from the support centre. Consequently, the variation of the support range allows to express the precision of the a-priori information. A necessary condition for consistency, however, is that the true elasticity remains within the support range. Only then it is possible that the increasing weight of the error probabilities in the objective function draws the parameter estimates to their true values as more observations become available.

The approach is analogous to the typical procedure in the framework of GME, but standard theoretical exposition (Golan et al. 1996) and agricultural economics applications (e.g. Lence and Miller 1998a and 1998b; Léon et al. 1999, Oude Lansink 1999; Zhang and Fan 2001) so far only employed direct restrictions on the parameter space to make the approach suitable for ill-posed and ill-conditioned problems. The restrictions on *functions* of parameters used here, however, are often more appropriate to incorporate available out of sample information.

After these technical remarks we come to the specific formulation of priors in our simulation experiments: The support point range for the elasticities is set to 4, so that a rather substantial variation of the estimated elasticities without strong penalties is possible. Two different support centres are considered. In one case they are equal to the true elasticities, in the other case they are shifted upwards by 0.3.

Table 1 QP-Model - RMSE of one estimated gross margin elasticity with different priors

	Sample size (T)				
Prior information	5	10	20	30	50
"without"	0.187	0.110	0.071	0.055	0.045
"true"	0.158	0.105	0.063	0.055	0.045
"false"	0.163	0.105	0.063	0.055	0.045

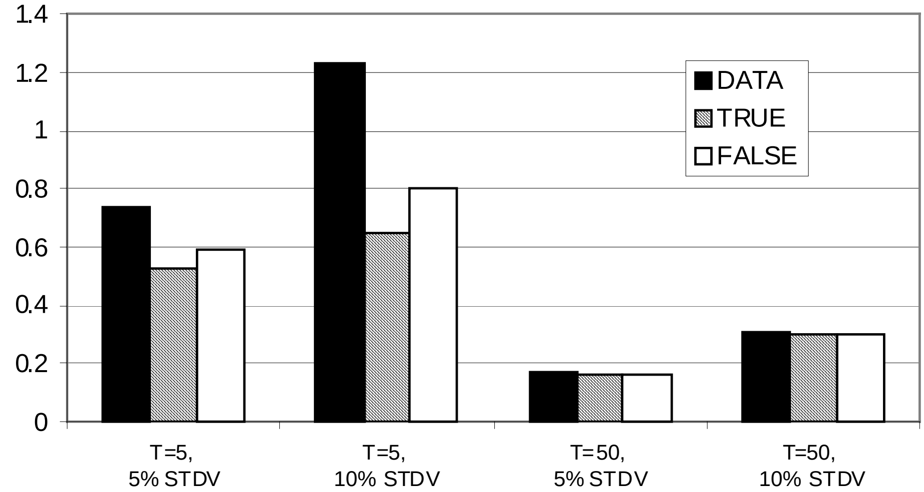
Source: Own calculations. The value of the true gross margin elasticity is 1.03.

Table 1 presents the RMSE of the gross margin elasticity of one output at different sample sizes. First we see, that the high variance of parameter estimates displayed in figure 1 is accompanied by a rather stable estimate of elasticities even with little data information. Nevertheless one can infer the general advantage of incorporating the prior information: The estimation error decreases for small sample sizes for both formulations of the priors, although the 'true' prior shows some advantage at the sample size of 5. Beyond a sample size of 20 no differences between the three variants exist and they all approach the true parameters with increasing sample sizes.

special case (multinomial model, see Golan et al. 1997).

The impact and usefulness of prior information is certainly also related to the noise in the data generation process. Figure 2 shows sums of root mean square errors across all three gross margin elasticities for two different standard deviations of the error terms (measured in percent of the mean land allocation). It becomes clear that the relative advantage of using priors at small sample sizes increases with the noise in the data generation process. However, for both versions, a sample size of 50 is enough to render the priors almost irrelevant for the quality of the estimates.

Figure 2 QP-model - SRMSE of estimated gross margin elasticities with different priors and noise components



Source: Own calculations.

Note, that the inclusion of prior information at small sample sizes can be seen as an intermediate approach between the calibration of the model to exogenous elasticities at some base year value and the estimation of model parameters with sufficient data information. Consequently, it allows to use at least the little data information available without jeopardising the 'plausibility' of the estimation results.

4.2 Input Allocation With Crop Specific Production Functions

In this section we want to look at a programming model which allocates variable and fixed inputs to different production activities with a functional representation of crop-specific production technology. The general form of the desired profit maximisation model is given by

$$\max_{x_{ik}, b_{ij}} Z = \sum_{i=1}^{N} p_i f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right) - \sum_{i=1}^{N}\sum_{k=1}^{K} q_k x_{ik}$$

(16) subject to

$$\sum_{i=1}^{N} b_{ij} = b_j \quad \left[\lambda_j\right]$$

where i, j , k are indices of outputs as well as fixed and variable inputs, respectively and $\boldsymbol{\theta}_i$ is a vector of technological parameters. Prices and allocated variable inputs are denoted as q_k and x_{ik}, b_{ij} and b_j represent allocated and available total quantities of the fixed inputs. The transformation of input to output quantities is given by

(17) $$y_i = f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right) .$$

The first order conditions comprise the resource constraints, the marginal value product conditions of variable inputs, and the shadow price equations of fixed factors:

(18) $$\sum_{i=1}^{N} b_{ij} = b_j , \quad \frac{\partial Z}{\partial x_{ik}} = p_i \frac{\partial f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right)}{\partial x_{ik}} - q_k = 0 , \quad \frac{\partial Z}{\partial b_{ij}} = p_i \frac{\partial f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right)}{\partial b_{ij}} - \lambda_j = 0 .$$

To solve this system of first order conditions for the input demand and output supply functions is very cumbersome if not impossible. Instead, we can use equations (17) and (18) directly as data constraints for estimating the unknowns $\boldsymbol{\theta}_i$ and λ_j. This implies a considerable advantage with respect to the choice of functional form as well as model complexity.

Again, we assume that the data generation process is disturbed by random errors around the endogenous model variables, here not only land allocations, but all input allocations x_{ik} and b_{ij} as well as supply quantities y_i. The corresponding errors e^x_{ikt}, e^b_{ijt}, and e^y_{it} for each observation are reparameterised as

(19) $$e^x_{ikt} = \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt} , \quad e^b_{ijt} = \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt} , \quad \text{and } e^y_{it} = \mathbf{v}^y_{i}\mathbf{w}^y_{it} ,$$

with the (1×2) vectors $\mathbf{v}^x_{ik}, \mathbf{v}^b_{ij}$, and $\mathbf{v}^y_{i}$ representing lower and upper support point and the (2×1) $\mathbf{w}^x_{ikt}$, $\mathbf{w}^b_{ijt}$, and $\mathbf{w}^y_{it}$ their corresponding probabilities for each observation. Adding indices for observations t = 1,...,T we obtain the complete GME program as

(20) $$\max_{\mathbf{w}^x_{ikt}, \mathbf{w}^b_{ijt}, \mathbf{w}^y_{it}, \boldsymbol{\theta}_i, \lambda_{jt}} H\left(\mathbf{w}^x_{ikt}, \mathbf{w}^b_{ijt}, \mathbf{w}^y_{it}\right) = -\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\mathbf{w}^x_{ikt}\,'\ln \mathbf{w}^x_{ikt} - \sum_{t=1}^{T}\sum_{j=1}^{M}\mathbf{w}^b_{ijt}\,'\ln \mathbf{w}^b_{ijt} - \sum_{t=1}^{T}\mathbf{w}^y_{it}\,'\ln \mathbf{w}^y_{it}\right]$$

subject to

(21) $$p_{it}\frac{\partial f_i\left(\left(x^o_{ikt} - \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt}\right), \left(b^o_{ijt} - \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt}\right) \mid \boldsymbol{\theta}_i\right)}{\partial x_{ik}} - q_{kt} = 0 ,$$

(22) $$\frac{\partial Z}{\partial b_{ij}} = p_{it}\frac{\partial f_i\left(\left(x^o_{ikt} - \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt}\right), \left(b^o_{ijt} - \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt}\right) \mid \boldsymbol{\theta}_i\right)}{\partial b_{ij}} - \lambda_{jt} = 0 .$$

(23) $$\left(y^o_{it} - \mathbf{v}^y_{i}\mathbf{w}^y_{it}\right) = f_{it}\left(\left(x^o_{ikt} - \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt}\right), \left(b^o_{jt} - \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt}\right) \mid \boldsymbol{\theta}_i\right) .$$

(24) $$\sum_{i=1}^{N}\left(b^{o}_{ijt}-\mathbf{v}^{b}_{ij}\mathbf{w}^{b}_{ijt}\right)=b_{jt},$$

(25) $$\sum_{s=1}^{S} w^{x}_{ikts}=1;\quad \sum_{s=1}^{S} w^{b}_{ijts}=1;\quad \sum_{s=1}^{S} w^{y}_{its}=1$$

Again, the data constraints have to be satisfied at estimated values of the endogenous variables calculated as the observed values minus the estimated errors.[12]

Before going to the set up and results of Monte Carlo simulations for this model we want to introduce some prior information also for this model to test its impact on the estimators accuracy. For this purpose we assume – as a variation from the previous model – that we have some information on the mean value of shadow prices of the fixed factors.[13] The GME approach needs to be modified by adding a constraint with the reparameterised mean shadow prices for – here two - fixed factors

(26) $$\mathbf{V}^{\lambda}\mathbf{w}^{\lambda}=\begin{bmatrix} v^{\lambda}_{11} & v^{\lambda}_{12} & 0 & 0 \\ 0 & 0 & v^{\lambda}_{21} & v^{\lambda}_{22} \end{bmatrix}\begin{bmatrix} w^{\lambda}_{11} \\ w^{\lambda}_{12} \\ w^{\lambda}_{21} \\ w^{\lambda}_{22} \end{bmatrix}=\begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}\lambda_{1t} \\ \frac{1}{T}\sum_{t=1}^{T}\lambda_{2t} \end{bmatrix}.$$

Also, the objective function is extended by the additional probabilities to read

(27) $$\max_{\mathbf{w}^{x}_{ikt},\mathbf{w}^{b}_{ijt},\mathbf{w}^{y}_{it},\mathbf{w}^{\lambda},\boldsymbol{\theta}_{i},\lambda_{jt}} H\left(\mathbf{w}^{x}_{ikt},\mathbf{w}^{b}_{ijt},\mathbf{w}^{y}_{it},\mathbf{w}^{\lambda}\right)=$$
$$-\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\mathbf{w}^{x}_{ikt}{}'\ln\mathbf{w}^{x}_{ikt}-\sum_{t=1}^{T}\sum_{j=1}^{M}\mathbf{w}^{b}_{ijt}{}'\ln\mathbf{w}^{b}_{ijt}-\sum_{t=1}^{T}\mathbf{w}^{y}_{it}{}'\ln\mathbf{w}^{y}_{it}\right]-\sum_{t=1}^{T}\mathbf{w}^{\lambda}{}'\ln\mathbf{w}^{\lambda}.$$

The functional form for the production technology chosen for the Monte Carlo simulations is the 'Constant Elasticity of Substitution' (CES) function, which distinguishes between two variable (chemicals and capital) and two fixed inputs (land and water).[14] This model structure is analogous to the PMP-CES approach by Howitt (1995b). However, the current model does not contain any additional non-linear cost terms in the objective function (and the estimation approach certainly does not require any determination of dual values of calibration constraints

---

[12] The introduction of error terms around the endogenous variables x_{ikt} und b_{ijt}, allow an estimation of input allocations consistent with the economic model. The presumed quality of 'observed' input allocations can be taken into account by varying the size of the support range.

[13] The employment of prior information on elasticities for this model is also possible despite the fact that an analytical expression for the elasticities might not be available. One can use discrete approximations based on additional 'artificial' constraints which are simply copies of the data constraints, but with systematically varied exogenous prices and variable 'simulated' output and input quantities. Although conceptually simple, the mathematical representation of this approach is considered too cumbersome for this paper.

[14] For the parameter values of the true model see the appendix. Additional parametric restrictions included during estimation in order to obtain a well defined production function are reported in the appendix as well.

from the 'step 1' of PMP). An additional difference is the requirement of decreasing returns to scale in order to allow for positive production levels of all crops.[15]

The data generation process adds normally distributed error terms to the optimal output and input quantities (with a standard deviation of 10% and 2% from the mean quantities) to obtain ‘observed’ allocations, where again the ‘incorrectly measured’ allocated quantities of the fixed inputs add up exactly to the available and known total quantities. For the simulation we distinguish again between a ‘true’ and a ‘false’ prior. The former defines supports for the shadow prices of land and water around the true values at the mean of the observations. The latter is defined by a support centre which is 10% below the true values. The size of supports is chosen to be 40% of the true mean shadow price. This is well above 5 standard deviations of the mean shadow prices across samples so that the support contains the true mean shadow price for both types of priors with almost certainty.

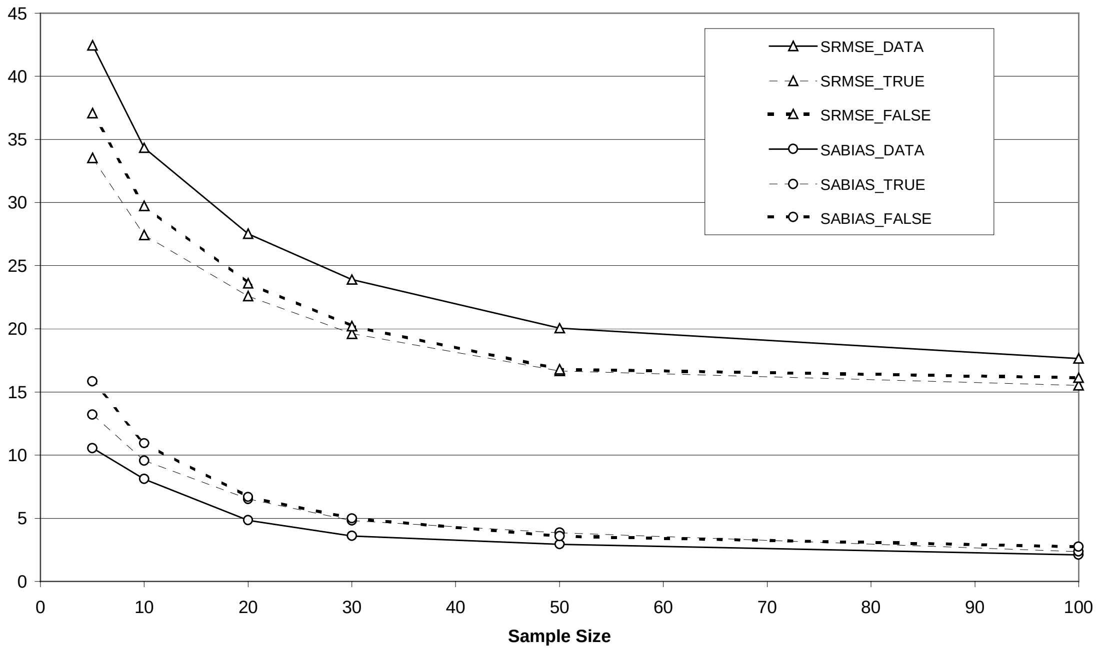
Figure 3 shows the absolute bias and the root mean square error as sums over the parameters of all three production functions (SABIAS and SRMSE). The decreasing SRMSE with increasing sample size hints at a consistent estimation approach. The employment of both types of prior information again reduces the SRMSE compared with the pure data case. The reduction is relatively modest compared to the priors on elasticities for the QP-model, but it is still relevant even for T=100. However, the difference between the ‘true’ and the ‘false’ prior is negligible from T=30 upwards. It is interesting to note, that the bias of the ‘true’ prior lies above the one for the ‘false’ prior. This can certainly happen in the case of a biased estimator and should be seen as a lucky ‘accident’. In fact, it can be shown that the result is reversed if we formulate the ‘false’ prior such that the centre of the supports lies above the true values.

Generally, the prior information for the shadow prices could also be formulated for every observation t instead of the mean of the shadow price, which might better reflect the type of data available (e.g. leasing rates for each observation). In this case, however, the number of associated probabilities in the objective function also increases with increasing observations. This may harm the convergence of the estimates to the true parameter values if the empirically unavoidable case occurs that the centres of the shadow price supports are not the true values. Additional simulations not reported here confirmed this hypothesis. This effect, however, could be compensated by including a factor in the objective function which continuously decreases the weight of the prior related probabilities with increasing sample size.

---

[15] Constant returns to scale (as in Howitt 1995b) would result in specialisation, since the maximum profit per unit of land in each activity would be independent of the land allocation. Consequently, the number of positive activity levels at the optimum could not be larger than the number of fixed factors as in a linear programming model.

Figure 3 SRMSE and SABIAS of parameter estimated with different prior information on shadow prices of fixed resources in the CES model



Source: Own calculations.

1.3 Allocation of Fixed Inputs with Crop Specific Profit Functions

The last example of a programming model keeps the general model structure of the last subsection with respect to assumptions on producer behaviour and crop specific technologies with allocable inputs, but employs duality concepts for the determination of variable output and input quantities. Going back to the case with only one fixed factor, the specification is equivalent to Guyomard et al. (1996) und Moro and Schokai (1999), who base their analysis on econometrically estimated systems of supply and explicit land allocation functions. On the one hand, we want to point out the full equivalence of our approach with respect to parameter estimation. On the other hand, we want to illustrate the advantages with respect to flexibility in the choice of functional form as well as the accommodated complexity of the model structure. The desired programming model is given by:

(28)
$$\max_{\mathbf{l}} Z = \sum_{i=1}^{N} \pi_i \left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i \right)$$
$$\text{subject to } \sum_{i=1}^{N} l_i = b \quad [\lambda]$$

where

(29)
$$\pi_i \left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i \right) = \max_{y_i, \mathbf{x}_i} \left[p_i y_i - \sum_{k=1}^{K} q_k x_{ik} \quad \text{subject to } \; y_i = f_i \left(x_{ik}, l_i \right) \right]$$

is a restricted profit function of crop i for a given land allocation l_i and $\boldsymbol{\theta}_i$ is now a vector of profit function parameters for product i. Model (28) distributes the available land b to the different production activities to maximise overall profit Z, where the profit of the single crops is determined by $\pi_i(p_i,\mathbf{q},l_i|\boldsymbol{\theta}_i)$. Consequently, the optimal land allocation is obtained if the marginal profits of land in each use are equal, i.e. if the first order conditions

$$(30) \qquad \frac{\partial \pi_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right)}{\partial l_i} - \lambda = 0$$

are satisfied. For some functional forms of $\pi_i(.)$ a solution of system (30) under additional consideration of the land constraint is possible and results in explicit land allocation equations depending on exogenous model parameters. Guyomard et al. describe the derivation based on normalised quadratic profit functions and estimate a system of land allocation equations and supply functions

$$(31) \qquad \frac{\partial \pi_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right)}{\partial p_i} = y_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right).$$

The resulting system is linear, but the regression coefficients have to satisfy non-linear constraints across equations. With our approach, no derivation of land allocation equations is necessary. Instead, the optimality conditions (30) are directly used in combination with (31) as data constraints of a GME approach analogous to the two cases in the previous subsections. As long as the same statistical model and econometric criterion is employed, the parameter estimates of this approach must be equal to the ones stemming from the estimation of the behavioural function, because of the mathematical equivalence of the data constraints. This could be confirmed on the basis of a GME and a non-linear least squares approach.

Why would we then want to estimate the model using the optimality conditions and subsequently employ a programming model for simulation purposes? The following points are to be mentioned: 1) The flexibility in choosing the functional form for $\pi_i(p_i,\mathbf{q},l_i)$ is greatly enhanced, because no closed form solution for land allocation functions is necessary. 2) For the same reason, a more complex model structure with more than one fixed factor or general constraints on land allocation (e.g. quotas, base areas...) is not a principal impediment anymore for the econometric estimation of the parameters. 3) The formulation of the resulting simulation model as an explicit optimisation model allows the flexible incorporation of additional relevant constraints on allocation for the simulation horizon without necessarily obstructing the structural validity of the estimation results.

Also for this model we performed simulation experiments based on an appropriate GME estimator. We mirror the approach by Guyomard et al. in the sense that we only employ data on supply quantities and land allocations, disregarding possible observations on allocated input quantities and the related input demand functions as data constraints. Reparameterising the errors of these endogenous variables of the programming model in the same way as for the CES production function model we can formulate the GME program for the estimation of the profit function parameters as

(32) $$\max_{\mathbf{w}_{it}^{l},\mathbf{w}_{it}^{y},\boldsymbol{\theta}_i,\lambda_t} \mathrm{H}\left(\mathbf{w}_{it}^{l},\mathbf{w}_{it}^{y}\right)=-\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sum_{j=1}^{M}\mathbf{w}_{it}^{l}\,'\ln\mathbf{w}_{it}^{l}-\sum_{t=1}^{T}\mathbf{w}_{it}^{y}\,'\ln\mathbf{w}_{it}^{y}\right]$$

subject to

(33) $$\frac{\partial\pi_i\left(p_{it},\mathbf{q}_t,\left(l_{it}^{o}-\mathbf{v}_i^{l}\mathbf{w}_{it}^{l}\right)\mid\boldsymbol{\theta}_i\right)}{\partial l_i}-\lambda=0\,,$$

(34) $$\frac{\partial\pi_i\left(p_i,\mathbf{q},\left(l_{it}^{o}-\mathbf{v}_i^{l}\mathbf{w}_{it}^{l}\right)\mid\boldsymbol{\theta}_i\right)}{\partial p_i}=\left(y_{it}^{o}-\mathbf{v}_i^{y}\mathbf{w}_{it}^{y}\right)$$

(35) $$\sum_{i=1}^{N}\left(b_{it}^{o}-\mathbf{v}_i^{l}\mathbf{w}_{it}^{l}\right)=b_t$$

(36) $$\sum_{s=1}^{S}w_{its}^{l}=1;\quad\sum_{s=1}^{S}w_{its}^{y}=1$$

Again, for different sample sizes, generated optimal supply quantities and land allocations were disturbed by normally distributed errors (with standard deviation of 10% and 2% of the mean variable values) and subsequent estimations without the use of prior information on parameters or functions thereof were executed. The results of the 1000 repetitions are given in table 2. The change of the different estimation errors (summed over all estimated parameters of the profit functions) indicates a consistent behaviour of the estimator.

Table 2 Profit function model – Monte Carlo results without prior information

	Sample size (T)						
Measures	4	5	10	20	30	50	100
SRMSE	2965	2888	1212	570	462	346	253
SABIAS	914	900	417	222	159	102	57
SSTD	2715	2672	1102	516	426	325	242

Source: Own calculations.

The high variance part of the mean squared error one more time suggests a large potential of plausible prior information – for example on elasticities – to improve the estimator's precision for small sample sizes. However, we refrain from a demonstration and focus instead on another issue of empirical relevance: Constraints on allocation such as the land constraint are frequently of the inequality type and across different observations those might be sometimes binding and sometimes not. As long as the data directly tell us whether a constraint is binding or not for each observation, this is straightforwardly handled by setting the shadow prices, a-priori, to zero for observations with non-binding constraints. But because of the noise in the data generation process, it is conceivable that the measured variable quantities give us the wrong impression. Apparently binding constraints might in fact be not binding for the true quantities and vice versa. In this case, we must allow the estimated or 'fitted' variable values to satisfy the constraints either in equality or inequality form. In principle, this can be easily accommodated by changing the land constraint (35) to the inequality type

(37) $$\sum_{i=1}^{N}\left(b_{it}^{o}-\mathbf{v}_i^{l}\mathbf{w}_{it}^{l}\right)\leq b_t ,$$

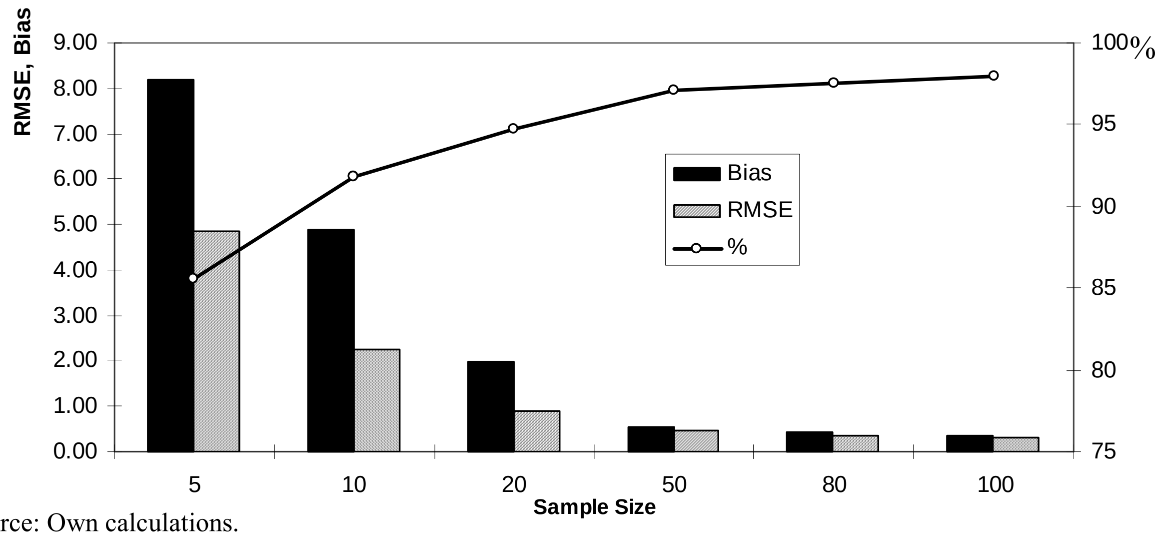
and adding the appropriate complementary slackness condition

(38) $$\sum_{i=1}^{N}\left(b_t-\left(b_{it}^{o}-\mathbf{v}_i^{l}\mathbf{w}_{it}^{l}\right)\right)\lambda_t=0 ,$$

to the GME approach. However, the numerical stability might be hampered with solvers based on gradient methods given the discontinuous nature of λ_t. In order to test this for the relatively simple example above we changed the data generation process for the simulation approach as follows:

First, the available mean level of land was increased in such a way that, on average, about 25% of the optimal solutions of the data generating programming model did not use all the land. Second, we did not enforce that the errors added to optimal land allocations sum to 0 which generally implies a non-zero difference between the sum of 'observed' land allocations and the available total quantity of land. Third, we use equations (37) and (38) instead of (35) in the GME approach. All other specifications of the simulation remained the same. The results are rather promising: As in the above experiments the SRMSE and SABIAS of $\boldsymbol{\theta}_i$ indicate a consistent behaviour of the estimation technique. However, to get a better insight of the technique's reflection of the mixed data generation process with non-binding and binding resources, in figure 4 we focus on the finite sample properties of the estimated dual values and on the ability of the approach to correctly identify the status of the constraint.

Figure 4 Bias and RMSE of the dual values and percentage of correctly identified status of the constraints as binding and non-binding



Source: Own calculations.

The RMSE of the dual values is calculated as the square of the difference between the means of the estimated shadow prices across all observations and the means of the true shadow prices for each repetition. Both the SABIAS and the RMSE diminish with increasing sample sizes. To provide further information we include - on the right axis - the percentage of correctly estimated observations concerning binding or non-binding status of the land constraint. It can be concluded that already for small sample sizes the estimation procedure is able to correctly identify binding and non-binding constraints at rates above 85% which is significantly higher than the cut-off value of 75%, i.e. the value obtained by assuming always binding constraints. With increasing sample size the rate almost approaches 100% indicating that the estimates converge to the true data generation process as the amount of data information increases.

5 Conclusions

The paper introduces a general approach for the estimation of programming models based on optimality conditions and shows the conceptual advantage compared to approaches based on PMP. The method simultaneously allows the specification of more complex models and a more flexible choice of functional form compared to previous estimation approaches of duality based behavioural functions with explicit allocation of fixed factors. The principle procedure and its functionality is demonstrated for three different examples of programming models. Monte Carlo simulations with a maximum entropy criterion indicate consistent behaviour of the estimator. In this context, the potential benefit from prior information on elasticities and shadow prices in situations with small sample sizes as well as the technical implementation could be shown. Last but not least, the approach proved its capability of estimating model parameters across binding and non-binding constraints in the data generation process.

Apart from different applications to large, ‘real world’ profit maximising programming models, many other directions for future research can be identified: extensions of the approach to multi-output production technologies with non-allocable variable factors or to expected utility models with risk might increase the empirical potential of these types of models by allowing for a higher level of differentiation. From an econometric methodology point of view, the GME approach leaves ample opportunities to improve upon current knowledge although its application is not a necessary requirement for the estimation of programming models in well posed situations: asymptotic properties of the estimator for non-linear models and easily applicable and valid test procedures are still missing. A more systematic investigation with respect to the formulation of prior information and their impact on estimation quality in small sample situations is also desirable.

References

Arfini, F. and Quirino, P. (1995). A positive mathematical programming model for regional analysis of agricultural policies. In Sotte, E. (ed), *The Regional Dimension in Agricultural Economics and Policies*, EAAE, Proceedings of the 40th Seminar, 26.-28. Juni 1995, Ancona, 17-35.

Barkaoui, A. and Butault, J.-P. (1999). Positive Mathematical Programming and Cereals and Oilseeds Supply within EU under Agenda 2000. Paper presented at the 9th European Congress of Agricultural Economists, Warsaw, August 1999.

Chambers, R.G. and Just, R.E. (1991). Estimating Multioutput Technologies, *American Journal of Agricultural-Economics* 71(4): 980-95.

Chirinko, R.S. (1993). Business Fixed Investment Spending: Modeling Strategies, Empirical Results, and Policy Implications, *Journal of Economic Literature* 31: 1875-1911.

Cypris, C. (2000). Positiv Mathematische Programmierung (PMP) im Agrarsektormodell RAUMIS. Dissertation, University of Bonn.

Golan, A., Judge, G. and Miller, D. (1996). *Maximum Entropy Econometrics*, Chichester UK: Wiley.

Golan, A., Judge, G. and Perloff, M. (1996). A Maximum Entropy Approach to Recovering Information from Multinomial Response Data, *Journal of the American Statistical Association* 91(434): 841-853.

Graindorge, C., de Frahan, B.H. and Howitt, R.E. (2001). Analysing the effects of Agenda 2000 Using a CES Calibrated Model of Belgian Agriculture. In Heckelei, T., Witzke, H.P. and Henrichsmeyer, W. (eds.): *Agricultural Sector Modelling and Policy Information Systems*. Proceedings of the 65th EAAE Seminar, March 29-31, 2000 at Bonn University, Vauk Verlag Kiel, 177-186.

Guyomard, H., Baudry, M. and Carpentier, A. (1996). Estimating crop supply response in the presence of farm programmes: application to the CAP. *European Review of Agricultural Economics* 23: 401-420.

Heckelei, T. and Britz, W. (2000). Positive Mathematical Programming with Multiple Data Points: A Cross-Sectional Estimation Procedure. *Cahiers d'Economie et Sociologie Rurales* 57: 28-50.

Helming, J.F.M., Peeters, L. and Veendendaal, P.J.J. (2001). Assessing the Consequences of Environmental Policy Scenarios in Flemish Agriculture. In Heckelei, T., Witzke, H.P. and Henrichsmeyer, W. (eds.): *Agricultural Sector Modelling and Policy Information Systems*. Proceedings of the 65th EAAE Seminar, March 29-31, 2000 at Bonn University, Vauk Verlag Kiel, 237-245.

Horner, G.L., Corman, J., Howitt, R.E., Carter, C.A. and MacGregor, R.J. (1992). The Canadian Regional Agriculture Model: Structure, Operation and Development. Agriculture Canada, Technical Report 1/92, Ottawa.

House, R.M. (1987). USMP Regional Agricultural Model. Washington DC: USDA. National Economics Division Report, ERS, 30.

Howitt, R.E. (1995a). Positive Mathematical Programming. *American Journal of Agricultural Economics* 77(2): 329-342.

Howitt, R.E. (1995b). A Calibration Method for Agricultural Economic Production Models. *Journal of Agricultural Economics* 46: 147-159.

Howitt, R.E., Gardner, B.D. (1986). Cropping Production and Resource Interrelationships among California Crops in Response to the 1985 Food Security Act. In *Impacts of Farm Policy and Technical Change on US and Californian Agriculture*, Davis (USA), 271-290.

Kasnakoglu, H. and Bauer, S. (1988). Concept and Application of an Agricultural Sector Model for Policy Analysis in Turkey. In S. Bauer and Henrichsmeyer, W. (eds.), *Agricultural Sector Modelling*. Kiel: Wissenschaftsverlag Vauck.

Lence, H.L. and Miller, D. (1998a). Estimation of Multioutput Production Functions with Incomplete Data: A Generalized Maximum Entropy Approach. *European Review of Agricultural Economics* 25: 188-209.

Lence, H.L. and Miller, D. (1998b). Recovering Output Specific Inputs from Aggregate Input Data: A Generalized Cross-Entropy Approach. *American Journal of Agricultural Economics* 80(4): 852-867.

LÉON Y., PEETERS, L., QUINQU, M. and SURRY, Y. (1999): The use of maximum entropy to estimate input-output coefficients from regional farm accounting data. *Journal of Agricultural Economics* 50: 425-439.

Mittelhammer, R.C. and Cardell, S. (2000). The Data-Constrained GME-Estimator of the GLM: Asymptotic Theory and Inference. Working Paper of the Department of Statistics, Washington State University, Pullman.

Mittelhammer, R.C., Judge, G.G. and Miller, D.J. (2000). Econometric Foundations. Cambridge University Press.

Moro, D. and Schokai, P. (1999). Modelling the CAP Arable Crop Regime in Italy: Degree of Decoupling and Impact of Agenda 2000. *Cahiers d'Economie et Sociologie Rurales* 53: 50-73.

Oude Lansink, A. (1999). Generalized Maximum Entropy and heterogeneous technologies. *European Review of Agricultural Economics* 26: 101-115.

Paris, Q. and Howitt, R.E. (1998). An Analysis of Ill-Posed Production Problems Using Maximum Entropy. *American Journal of Agricultural Economics* 80(1): 124-138.

Paris, Q. and Howitt, R.E. (2001). The Multi-Input Multi-Output Symmetric Positive Equilibrium Problem. In Heckelei, T., Witzke, H.P. and Henrichsmeyer, W. (eds.): *Agricultural Sector Modelling and Policy Information Systems*. Proceedings of the 65^{th} EAAE Seminar, March 29-31, 2000 at Bonn University, Vauk Verlag Kiel, 88-100.

Preckel, P.V. (2001). Least Squares and Entropy: A Penalty Function Perspective. *American Journal of Agricultural Economics* 83(2): 366-377.

Schmitz, H.-J.(1994). Entwicklungsperspektiven der Landwirtschaft in den neuen Bundesländern - Regionaldifferenzierte Simulationsanalysen Alternativer Agrarpolitischer Szenarien. Studien zur Wirtschafts- und Agrarpolitik, Witterschlick/Bonn: M. Wehle.

Zhang, X. and Fan, S. (2001). Estimating Crop-Specific Production Technologies in Chinese Agriculture: A Generalized Maximum Entropy Approach. *American Journal of Agricultural Economics* 83(2): 378-388.

APPENDIX

A1. Basic data set

The following data set from Howitt (1995b) provided the differentiation of all presented models with respect to outputs and inputs. Variable quantities are the means of random variables and values of fixed variables for all Monte Carlo simulations.

Table A1: Base year data on California agriculture

			Input Allocation			
Crop	Price ($/bu)	Yield (bu/acre)	Land (10^6 acres)	Water (10^6 acre ft)	Capital (Index)	Chemicals (Index)
Cotton	2.924	220	1.49	4.47	3.96	2.64
Wheat	2.98	85	0.62	1.14	1.98	1.32
Rice	7.09	70.1	0.54	3.08	2.94	1.96
		Variable Input Prices ($)		10	10	10
		Resource Constraints	2.65	8.69		

Source: Based on Howitt (1995b)

A2. Information on Monte Carlo simulation with QP-model

True parameter values of **Q**:

$$\mathbf{Q} = \begin{bmatrix} 500 & -20 & -10 \\ -10 & 60 & -2 \\ -10 & -2 & 200 \end{bmatrix}.$$

A3 Information on Monte Carlo Simulation with CES-model

Functional form of production functions and true parameter values:

$$y_i = f\left(x_{ik}, b_{ij} \mid \theta_i\right) = \alpha_i \left(\sum_{k=1}^{2} \beta_{ik} x_{ik}^{\gamma_i} + \sum_{j=3}^{4} \beta_{ij} b_{ij}^{\gamma_i} \right)^{\nu_i/\gamma_i}$$

where

$$\boldsymbol{\theta}_1 = \begin{bmatrix} \alpha_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \gamma_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} 200 \\ 0.2 \\ 0.1 \\ 0.6 \\ 0.1 \\ -0.25 \\ 0.6 \end{bmatrix}; \quad \boldsymbol{\theta}_2 = \begin{bmatrix} \alpha_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \gamma_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} 100 \\ 0.1 \\ 0.1 \\ 0.7 \\ 0.1 \\ -0.25 \\ 0.8 \end{bmatrix}; \quad \boldsymbol{\theta}_3 = \begin{bmatrix} \alpha_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \gamma_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} 50 \\ 0.1 \\ 0.1 \\ 0.5 \\ 0.3 \\ -0.25 \\ 0.8 \end{bmatrix}$$

Parametric restrictions enforced during estimation:

$$\alpha_i \geq 0; \quad 0 \leq \beta_{ij} \leq 1; \quad \sum_{j=1}^{4} \beta_{ij} = 1 \;;$$

$$\sigma_i = \frac{1}{1-\gamma_i} \geq 0 \;; \quad 0 \leq \nu_i \leq 1 \;.$$

A4. Information on Monte Carlo simulation with profit function model

Functional form of the profit functions and true parameter values:

$$\pi_i(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i) = \alpha_{0i} + \alpha_{1i}\frac{p_i}{q_2} + \alpha_{2i}\frac{q_1}{q_2} + \alpha_{3i} l_i + 0.5\beta_{1i}\left(\frac{p_i}{q_2}\right)^2 + 0.5\beta_{2i}\left(\frac{q_1}{q_2}\right)^2 + 0.5\beta_{3i}\left(l_i\right)^2$$

$$+ \gamma_{1i}\frac{p_i q_1}{q_2^2} + \gamma_{2i}\frac{p_i}{q_2} l_i + \gamma_{3i}\frac{q_1}{q_2} l_i$$

where

$$\boldsymbol{\theta}_1 = \begin{bmatrix} \alpha_{01} \\ \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \beta_{11} \\ \beta_{21} \\ \beta_{31} \\ \gamma_{11} \\ \gamma_{21} \\ \gamma_{31} \end{bmatrix} = \begin{bmatrix} -67.6571 \\ 115.023 \\ 1.914 \\ 87.836 \\ 24.854 \\ 1.167 \\ -60.321 \\ -9.391 \\ 144.229 \\ -2.883 \end{bmatrix}; \quad \boldsymbol{\theta}_2 = \begin{bmatrix} \alpha_{02} \\ \alpha_{12} \\ \alpha_{22} \\ \alpha_{32} \\ \beta_{12} \\ \beta_{22} \\ \beta_{32} \\ \gamma_{12} \\ \gamma_{22} \\ \gamma_{32} \end{bmatrix} = \begin{bmatrix} -16.2746 \\ -34.130 \\ -0.176 \\ 49.817 \\ 23.552 \\ 0.607 \\ -82.432 \\ -4.232 \\ 135.553 \\ -1.855 \end{bmatrix}; \quad \boldsymbol{\theta}_3 = \begin{bmatrix} \alpha_{03} \\ \alpha_{13} \\ \alpha_{23} \\ \alpha_{33} \\ \beta_{13} \\ \beta_{23} \\ \beta_{33} \\ \gamma_{13} \\ \gamma_{23} \\ \gamma_{33} \end{bmatrix} = \begin{bmatrix} -8.7735 \\ 4.656 \\ -0.650 \\ 27.335 \\ 6.115 \\ 0.882 \\ -53.635 \\ -2.611 \\ 58.284 \\ -2.446 \end{bmatrix}.$$

Parametric restrictions enforced during estimation:

$\beta_{1i} > 0$; $\beta_{2i} > 0$; $\gamma_{i2} > 0$; $\beta_{3i} < 0$; $\gamma_{i1} < 0$;

Parameters not estimated (only appear in profit function or input demand functions):

α_{0i}, α_{2i} , β_{2i}

Parameters not identified relative to shadow price of land (same as in Guyomard et al., where those parameters are part of the composite estimated regression coefficients in the land allocation equations) and therefore fixed at true values:

α_{31}, γ_{31}.